1. You drop a 7cm long piece of mechanical pencil lead on the floor. A bully takes the lead and breaks it at a random point into two pieces. A piece of lead is unusable if it is 2cm or shorter. If the expected value of the number of usable pieces afterwards is \( \frac{m}{n} \) for relatively prime positive integers \( m \) and \( n \), compute \( 100m + n \).

2. Let \( ABC \) be an equilateral triangle. Denote by \( D \) the midpoint of \( BC \), and denote the circle with diameter \( AD \) by \( \Omega \). If the region inside \( \Omega \) and outside \( \triangle ABC \) has area \( 800\pi - 600\sqrt{3} \), find the length of \( AB \).

3. In land of Nyemo, the unit of currency is called a quack. The citizens use coins that are worth 1, 5, 25, and 125 quacks. How many ways can someone pay off 125 quacks using these coins?

4. Let \( S \) be the set of integers which are both a multiple of 70 and a factor of 630,000. A random element \( c \) of \( S \) is selected. If the probability that there exists an integer \( d \) with \( \gcd(c, d) = 70 \) and \( \text{lcm}(c, d) = 630,000 \) is \( \frac{m}{n} \) for some relatively prime integers \( m \) and \( n \), compute \( 100m + n \).

5. Triangle \( ABC \) has sidelengths \( AB = 14, BC = 15, \) and \( CA = 13 \). We draw a circle with diameter \( AB \) such that it passes \( BC \) again at \( D \) and passes \( CA \) again at \( E \). If the circumradius of \( \triangle CDE \) can be expressed as \( \frac{m}{n} \) where \( m, n \) are coprime positive integers, determine \( 100m + n \).

6. Let \( N = 10^6 \). For which integer \( a \) with \( 0 \leq a \leq N - 1 \) is the value of \[ \binom{N}{a+1} - \binom{N}{a} \] maximized?

7. Find the sum of all integers \( n \) with \( 2 \leq n \leq 999 \) and the following property: if \( x \) and \( y \) are randomly selected without replacement from the set \( \{1, 2, \ldots, n\} \), then \( x + y \) is even with probability \( p \), where \( p \) is the square of a rational number.

8. Let \( a, b, c, d \) be complex numbers satisfying
\[
\begin{align*}
5 &= a + b + c + d \\
125 &= (5-a)^4 + (5-b)^4 + (5-c)^4 + (5-d)^4 \\
1205 &= (a+b)^4 + (b+c)^4 + (c+d)^4 + (d+a)^4 + (a+c)^4 + (b+d)^4 \\
25 &= a^4 + b^4 + c^4 + d^4
\end{align*}
\]
Compute \( abcd \).