Calculators and other computational aids are NOT allowed.

Problem 1. Let $x, y$ be positive real numbers. If

$$129 - x^2 = 195 - y^2 = xy,$$

then $x = \frac{m}{n}$ for relatively prime positive integers $m, n$. Find $100m + n$.

Problem 2. Trapezoid $ABCD$ is an isosceles trapezoid with $AD = BC$. Point $P$ is the intersection of the diagonals $AC$ and $BD$. If the area of $\triangle ABP$ is 50 and the area of $\triangle CDP$ is 72, what is the area of the entire trapezoid?

Problem 3. How many triples of integers $(a, b, c)$ with $-10 \leq a, b, c \leq 10$ satisfy

$$a^2 + b^2 + c^2 = (a + b + c)^2?$$

Problem 4. How many subsets of the set \{1, 2, \ldots, 11\} have median 6?

Problem 5. Compute the only element of the set

$$\{1, 2, 3, 4, \ldots\} \cap \left\{ \frac{404}{r^2 - 4} \mid r \in \mathbb{Q} \setminus \{-2, 2\} \right\}.$$

Problem 6. Suppose $a, b, c$ are positive integers such that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} - \frac{524}{abc} = \frac{b}{a} + \frac{c}{b} + \frac{a}{c} - \frac{518}{abc} = 1.$$

Find $a^2 + b^2 + c^2$.

Problem 7. Call a pair of integers $(a, b)$ primitive if there exists a positive integer $\ell$ such that $(a + bi)^\ell$ is real. Find the smallest positive integer $n$ such that less than 1% of the pairs $(a, b)$ with $0 \leq a, b \leq n$ are primitive.

Problem 8. Let $ABC$ be a triangle with $BC = 49$ and circumradius 25. Suppose that the circle centered on $BC$ that is tangent to $AB$ and $AC$ is also tangent to the circumcircle of $ABC$. Then

$$\frac{AB \cdot AC}{-BC + AB + AC} = \frac{m}{n}$$

where $m$ and $n$ are relatively prime positive integers. Compute $100m + n$.

Time limit: 40 minutes
Maximum score is 56 points