NIMO Monthly Contest

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Contest 12 8:00 PM – 8:40 PM ET

February 24, 2014

1. You drop a 7cm long piece of mechanical pencil lead on the floor. A bully takes the lead and breaks it at a random point into two pieces. A piece of lead is unusable if it is 2cm or shorter. If the expected value of the number of usable pieces afterwards is $\frac{m}{n}$ for relatively prime positive integers m and n, compute $100m + n$.

2. Let ABC be an equilateral triangle. Denote by D the midpoint of $\overline{BC}$, and denote the circle with diameter $\overline{AD}$ by Ω. If the region inside Ω and outside $\triangle ABC$ has area $800\pi - 600\sqrt{3}$, find the length of AB.

3. In land of Nyemo, the unit of currency is called a *quack*. The citizens use coins that are worth 1, 5, 25, and 125 quacks. How many ways can someone pay off 125 quacks using these coins?

4. Let S be the set of integers which are both a multiple of 70 and a factor of 630,000. A random element c of S is selected. If the probability that there exists an integer d with $\gcd(c, d) = 70$ and $\operatorname{lcm}(c, d) = 630{,}000$ is $\frac{m}{n}$ for some relatively prime integers m and n, compute $100m + n$.

5. Triangle ABC has sidelengths $AB = 14$, $BC = 15$, and $CA = 13$. We draw a circle with diameter $\overline{AB}$ such that it passes $\overline{BC}$ again at D and passes $\overline{CA}$ again at E. If the circumradius of $\triangle CDE$ can be expressed as $\frac{m}{n}$ where m, n are coprime positive integers, determine $100m+n$.

6. Let $N = 10^6$. For which integer a with $0 \le a \le N - 1$ is the value of
$$\binom{N}{a+1} - \binom{N}{a}$$
maximized?

7. Find the sum of all integers n with $2 \le n \le 999$ and the following property: if x and y are randomly selected without replacement from the set $\{1, 2, \ldots, n\}$, then $x + y$ is even with probability p, where p is the square of a rational number.

8. Let a, b, c, d be complex numbers satisfying
$$\begin{aligned} 5 &= a + b + c + d \\ 125 &= (5-a)^4 + (5-b)^4 + (5-c)^4 + (5-d)^4 \\ 1205 &= (a+b)^4 + (b+c)^4 + (c+d)^4 + (d+a)^4 + (a+c)^4 + (b+d)^4 \\ 25 &= a^4 + b^4 + c^4 + d^4 \end{aligned}$$
Compute $abcd$.