NIMO

internetolympiad.org
National Internet Math Olympiad

Contest **32**

May 11, 2017
8:00 PM – 8:40 PM ET

Calculators and other computational aids are NOT allowed.

Problem 1. In the diagram below, how many rectangles can be drawn using the grid lines which contain none of the letters N, I, M, O?

		I		
	N		M	
		O		

Problem 2. Find the smallest positive integer N for which N is divisible by 19, and when the digits of N are read in reverse order, the result (after removing any leading zeroes) is divisible by 36.

Problem 3. In rectangle $ABCD$ with center O, $AB = 10$ and $BC = 8$. Circle γ has center O and lies tangent to $\overline{AB}$ and $\overline{CD}$. Points M and N are chosen on $\overline{AD}$ and $\overline{BC}$, respectively; segment MN intersects γ at two distinct points P and Q, with P between M and Q. If $MP : PQ : QN = 3 : 5 : 2$, then the length MN can be expressed in the form $\sqrt{a} - \sqrt{b}$, where a, b are positive integers. Find $100a + b$.

Problem 4. For each positive integer n, let r_n be the smallest positive root of the equation $x^n = 7x - 4$. There are positive real numbers a, b, and c such that

$$\lim_{n\to\infty} a^n(r_n - b) = c.$$

If $100a + 10b + c = \frac{p}{7}$ for some integer p, find p.

Problem 5. Let $p = 2017$ be a prime number. Let E be the expected value of the expression

$$3 \square 3 \square 3 \square \cdots \square 3 \square 3$$

where there are $p+3$ threes and $p+2$ boxes, and one of the four arithmetic operations $\{+, -, \times, \div\}$ is uniformly chosen at random to replace each of the boxes. If $E = \frac{m}{n}$, where m and n are relatively prime positive integers, find the remainder when $m + n$ is divided by p.

Time limit: 40 minutes
Maximum score is 56 points