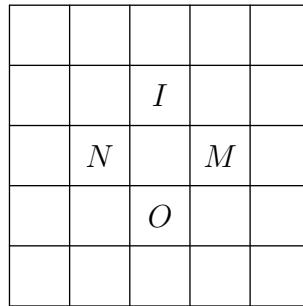


May 11, 2017  
8:00 PM – 8:40 PM ET

**Calculators and other computational aids are NOT allowed.**

**Problem 1.** In the diagram below, how many rectangles can be drawn using the grid lines which contain none of the letters  $N$ ,  $I$ ,  $M$ ,  $O$ ?



**Problem 2.** Find the smallest positive integer  $N$  for which  $N$  is divisible by 19, and when the digits of  $N$  are read in reverse order, the result (after removing any leading zeroes) is divisible by 36.

**Problem 3.** In rectangle  $ABCD$  with center  $O$ ,  $AB = 10$  and  $BC = 8$ . Circle  $\gamma$  has center  $O$  and lies tangent to  $\overline{AB}$  and  $\overline{CD}$ . Points  $M$  and  $N$  are chosen on  $\overline{AD}$  and  $\overline{BC}$ , respectively; segment  $MN$  intersects  $\gamma$  at two distinct points  $P$  and  $Q$ , with  $P$  between  $M$  and  $Q$ . If  $MP : PQ : QN = 3 : 5 : 2$ , then the length  $MN$  can be expressed in the form  $\sqrt{a} - \sqrt{b}$ , where  $a, b$  are positive integers. Find  $100a + b$ .

**Problem 4.** For each positive integer  $n$ , let  $r_n$  be the smallest positive root of the equation  $x^n = 7x - 4$ . There are positive real numbers  $a, b$ , and  $c$  such that

$$\lim_{n \rightarrow \infty} a^n(r_n - b) = c.$$

If  $100a + 10b + c = \frac{p}{7}$  for some integer  $p$ , find  $p$ .

**Problem 5.** Let  $p = 2017$  be a prime number. Let  $E$  be the expected value of the expression

$$3 \square 3 \square 3 \square \dots \square 3 \square 3$$

where there are  $p+3$  threes and  $p+2$  boxes, and one of the four arithmetic operations  $\{+, -, \times, \div\}$  is uniformly chosen at random to replace each of the boxes. If  $E = \frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers, find the remainder when  $m + n$  is divided by  $p$ .

Time limit: 40 minutes  
Maximum score is 56 points