April 7, 2017
8:00 PM – 8:40 PM ET

Calculators and other computational aids are NOT allowed.

Problem 1. Four people were guessing the number, N, of jellybeans in a jar. No two guesses were equally close to N. The closest guess was 80 jellybeans, the next closest guess was 60 jellybeans, followed by 49 jellybeans, and the furthest guess was 125 jellybeans. Find the sum of all possible values for N.

Problem 2. Suppose that

$$\operatorname{lcm}(1024, 2016) = \operatorname{lcm}(1024, 2016, x_1, x_2, \ldots, x_n),$$

with x_1, x_2, $\cdots$, x_n are distinct postive integers. Find the maximum value of n.

Problem 3. Let $ABCD$ be a cyclic quadrilateral with circumradius $100\sqrt{3}$ and $AC = 300$. If $\angle DBC = 15^\circ$, then find AD^2.

Problem 4. Isabella has a sheet of paper in the shape of a right triangle with sides of length 3, 4, and 5. She cuts the paper into two pieces along the altitude to the hypotenuse, and randomly picks one of the two pieces to discard. She then repeats the process with the other piece (since it is also in the shape of a right triangle), cutting it along the altitude to its hypotenuse and randomly discarding one of the two pieces once again, and continues doing this forever. As the number of iterations of this process approaches infinity, the total length of the cuts made in the paper approaches a real number l. Compute $[\mathbb{E}(l)]^2$, that is, the square of the expected value of l.

Problem 5. Triangle ABC has side lengths $AB = 13$, $BC = 14$, and $CA = 15$. Points D and E are chosen on AC and AB, respectively, such that quadrilateral $BCDE$ is cyclic and when the triangle is folded along segment DE, point A lies on side BC. If the length of DE can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n, find $100m + n$.

Problem 6. Define $f(x) = x^2 - 45x + 21$. Find the sum of all positive integers n with the following property: there is exactly one integer i in the set $\{1, 2, \ldots, n\}$ such that n divides $f(i)$.

Problem 7. Let the function $f(x) = \lfloor x \rfloor \{x\}$. Compute the smallest positive integer n such that the graph of $f(f(f(x)))$ on the interval $[0, n]$ is the union of 2017 or more line segments.

Problem 8. Let N be the number of integer sequences $a_1, a_2, \ldots, a_{2^{16}-1}$ satisfying

$$0 \le a_{2k+1} \le a_k \le a_{2k+2} \le 1$$

for all $1 \le k \le 2^{15} - 1$. Find the number of positive integer divisors of N.

Time limit: 40 minutes
Maximum score is 56 points