NIMO internetolympiad.org
National Internet Math Olympiad
Contest 28

October 18, 2016
8:00 PM – 8:40 PM ET

Calculators and other computational aids are NOT allowed.

Problem 1. Let x, y be positive real numbers. If

$$129 - x^2 = 195 - y^2 = xy,$$

then $x = \frac{m}{n}$ for relatively prime positive integers m, n. Find $100m + n$.

Problem 2. Trapezoid $ABCD$ is an isosceles trapezoid with $AD = BC$. Point P is the intersection of the diagonals AC and BD. If the area of $\triangle ABP$ is 50 and the area of $\triangle CDP$ is 72, what is the area of the entire trapezoid?

Problem 3. How many triples of integers (a, b, c) with $-10 \le a, b, c \le 10$ satisfy

$$a^2 + b^2 + c^2 = (a + b + c)^2?$$

Problem 4. How many subsets of the set $\{1, 2, \ldots, 11\}$ have median 6?

Problem 5. Compute the only element of the set

$$\{1, 2, 3, 4, \ldots\} \cap \left\{ \frac{404}{r^2 - 4} \,\middle|\, r \in \mathbb{Q} \backslash \{-2, 2\} \right\}.$$

Problem 6. Suppose a, b, and c are positive integers such that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} - \frac{524}{abc} = \frac{b}{a} + \frac{c}{b} + \frac{a}{c} - \frac{518}{abc} = 1.$$

Find $a^2 + b^2 + c^2$.

Problem 7. Call a pair of integers (a, b) *primitive* if there exists a positive integer ℓ such that $(a + bi)^\ell$ is real. Find the smallest positive integer n such that less than 1% of the pairs (a, b) with $0 \le a, b \le n$ are primitive.

Problem 8. Let ABC be a triangle with $BC = 49$ and circumradius 25. Suppose that the circle centered on BC that is tangent to AB and AC is also tangent to the circumcircle of ABC. Then

$$\frac{AB \cdot AC}{-BC + AB + AC} = \frac{m}{n}$$

where m and n are relatively prime positive integers. Compute $100m + n$.

Time limit: 40 minutes
Maximum score is 56 points