NIMO

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Contest **26**

May **2016**

May 12, 2016
8:00 PM – 8:40 PM ET

Calculators and other computational aids are NOT allowed.

Problem 1. Three congruent circles of radius 2 are drawn in the plane so that each circle passes through the centers of the other two circles. The region common to all three circles has a boundary consisting of three congruent circular arcs. Let K be the area of the triangle whose vertices are the midpoints of those arcs. If $K = \sqrt{a} - b$ for positive integers a, b, find $100a + b$.

Problem 2. Find the greatest positive integer n such that 2^n divides

$$\operatorname{lcm}\left(1^1, 2^2, 3^3, \ldots, 2016^{2016}\right).$$

Problem 3. A round-robin tournament has six competititors. Each round between two players is equally likely to result in a win for a given player, a loss for that player, or a tie. The results of the tournament are *nice* if for all triples of distinct players (A, B, C),

1. If A beat B and B beat C, then A also beat C;
2. If A and B tied, then either C beat both A and B, or C lost to both A and B.

The probability that the results of the tournament are *nice* is $p = \frac{m}{n}$, for coprime positive integers m and n. Find m.

Problem 4. Triangle ABC has $AB = 13$, $BC = 14$, and $CA = 15$. Let ω_A, ω_B and ω_C be circles such that ω_B and ω_C are tangent at A, ω_C and ω_A are tangent at B, and ω_A and ω_B are tangent at C. Suppose that line AB intersects ω_B at a point $X \neq A$ and line AC intersects ω_C at a point $Y \neq A$. If lines XY and BC intersect at P, then $\frac{BC}{BP} = \frac{m}{n}$ for coprime positive integers m and n. Find $100m + n$.

Problem 5. The equation $x^3 - 3x^2 - 7x - 1 = 0$ has three distinct real roots a, b, and c. If

$$\left(\frac{1}{\sqrt[3]{a} - \sqrt[3]{b}} + \frac{1}{\sqrt[3]{b} - \sqrt[3]{c}} + \frac{1}{\sqrt[3]{c} - \sqrt[3]{a}}\right)^2 = \frac{p\sqrt[3]{q}}{r}$$

where p, q, r are positive integers such that $\gcd(p, r) = 1$ and q is not divisible by the cube of a prime, find $100p + 10q + r$.

Time limit: 40 minutes
Maximum score is 56 points