3rd Annual NIMO April Round

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April 1, 2015

Welcome to the 2015 April Fun Round. This one's shorter and more accessible than the ones we've had in the past, with the intention that you can reasonably work on it during AP Chemistry or something.

You may use any aids such as calculators, computers, Wikipedia, etc. You are also free to collaborate with other students (spread the fun!) but we ask that you do so only in private mediums to give everyone a chance at the problems (say, email and chat as opposed to public forums).

You have the whole day, and each answer is a positive integer. Moreover, note that you are allowed five entries per problem instead of the usual three.

Contents

1 (2 points) Distributing Candy

A teacher has a class consisting of 243 hungry schoolchildren sitting in a circle. She wishes to distribute at most N pieces of candy to the schoolchildren, so that

- Every schoolchild receives at most $\lceil N/2 \rceil$ pieces of candy.
- Some schoolchild does not receive any candy.
- No eight consecutive schoolchildren all obtain candy.

How many ways are there to do so for $N = 1$?

2 (1 point) Distributing Less Candy

A teacher has a class consisting of 243 hungry schoolchildren sitting in a circle. She wishes to distribute at most N pieces of candy to the schoolchildren, so that

- Every schoolchild receives at most $\lceil N/2 \rceil$ pieces of candy.
- Some schoolchild does not receive any candy.
- No eight consecutive schoolchildren all obtain candy.

How many ways are there to do so for $N = 0$?

3 (3 points) Engineer's Induction II

Assume this test is extended to nine questions. What should the total point value of the test be?

4 (4 points) Orthocenters

Let $\triangle A_1A_2A_3$ be a triangle such that $A_1A_2 = 134$, $A_2A_3 = 145$, and $A_1A_3 = 156$. Define a sequence of coplanar points $\{A_i\}_{i=1}^{\infty}$ such that for each integer $k \geq 4$, A_k is the orthocenter of $\triangle A_{k-1}A_{k-2}A_{k-3}$. Find the distance between the points A_i and A_j, where

$$i = 2011^{2012^{2013}} \quad \text{and} \quad j = 2011^{2013^{2015}}.$$

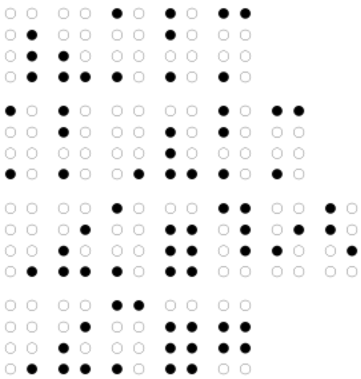
5 (7 points) Discipline

A mathematician, computer scientist, engineer have a function f. However, they don't agree on values this function returns. Here are some sample values.

- The mathematician thinks that $f(18.701) \approx 2.929$ and $f(20.15) \approx 3.003$.
- The computer scientist thinks that $f(6.001) \approx 2.585$ and $f(18.940) \approx 4.243$.
- The engineer thinks that $f(8.022) \approx 0.904$ and $f(2015) = x$.

To the nearest integer, what is $10x$?

6 (11 points) GS8



7 (18 points) Not Lagrange Murderpliers

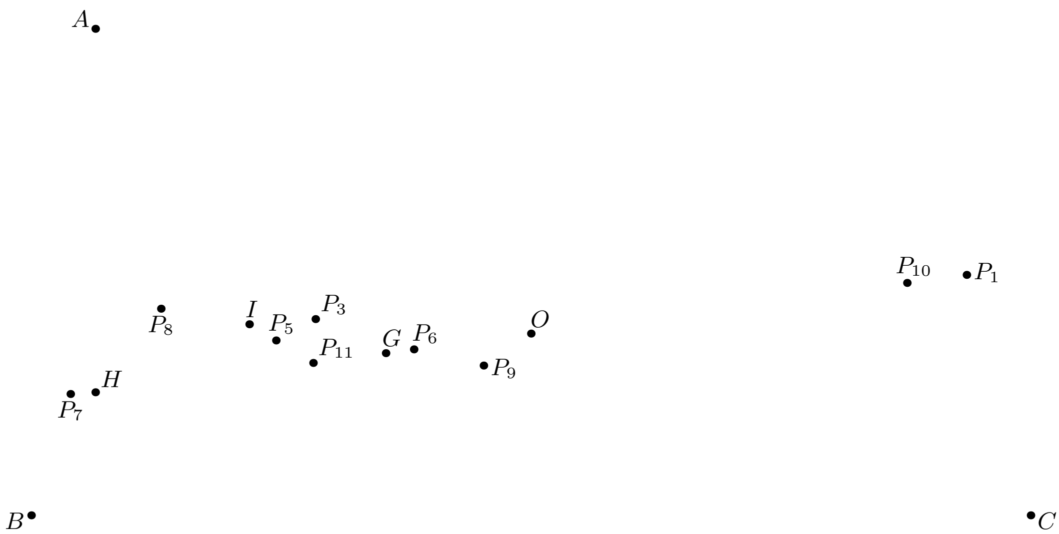
Let a, b, c be real numbers which satisfy

$$\begin{aligned}-23603 = {} & 98a^4 + 212a^3b + 260a^3c - 1504a^3 + 294a^2b^2 \\ & + 432a^2bc - 2652a^2b + 294a^2c^2 - 3060a^2c + 8772a^2 \\ & + 260ab^3 + 432ab^2c - 2676ab^2 + 432abc^2 - 3888abc \\ & + 11364ab + 212ac^3 - 2556ac^2 + 12420ac - 23176a + 98b^4 \\ & + 212b^3c - 1240b^3 + 294b^2c^2 - 2244b^2c + 6276b^2 + 260bc^3 - 2364bc^2 \\ & + 9348bc - 16816b + 98c^4 - 1144c^3 + 6036c^2 - 17680c.\end{aligned}$$

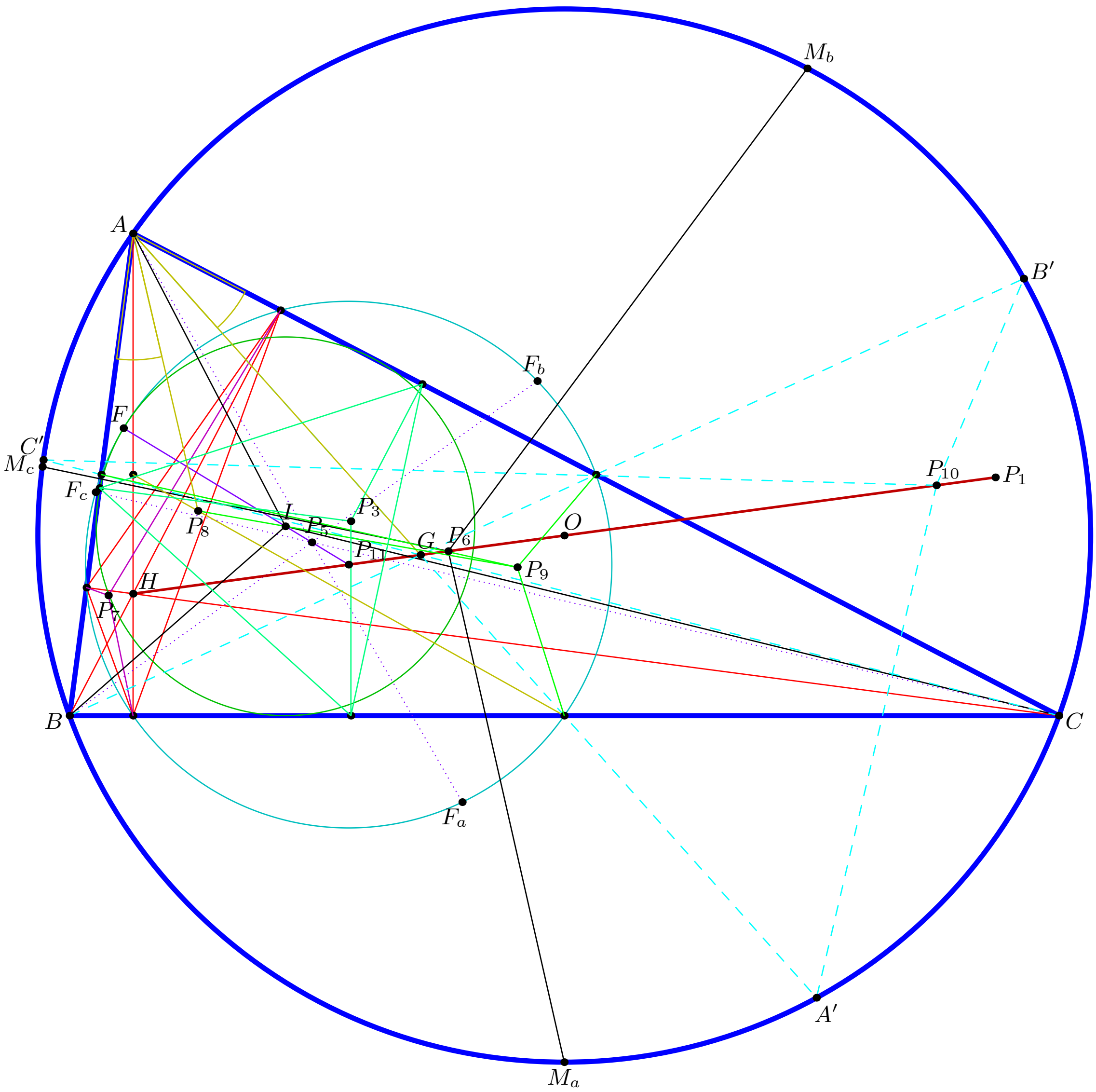
What is the largest possible value of $a + b + c + \frac{1}{2}$?

8 (29 points) GeoGuessr

In what follows, $5 \to$ E, $6 \to$ F, and so on.



For your sanity, here is another version with some auxiliary stuff drawn in, although you *strictly speaking* don't need them.



Not pictured, because they lie outside the triangle.

- P_2, which is the inverse of G with respect to O,
- P_4, which is the harmonic conjugate of P_3 with respect to O and P_8.