NIMO

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Contest 19

May 2015

May 19, 2015
8:00 PM – 8:40 PM ET

Calculators and other computational aids are not allowed.

Problem 1. Let Ω_1 and Ω_2 be two circles in the plane. Suppose the common external tangent to Ω_1 and Ω_2 has length 2017 while their common internal tangent has length 2009. Find the product of the radii of Ω_1 and Ω_2.

Problem 2. Consider the set S of the eight points (x, y) in the Cartesian plane satisfying $x, y \in \{-1, 0, 1\}$ and $(x, y) \neq (0, 0)$. How many ways are there to draw four segments whose endpoints lie in S such that no two segments intersect, even at endpoints?

Problem 3. Let O, A, B, and C be points in space such that $\angle AOB = 60°$, $\angle BOC = 90°$, and $\angle COA = 120°$. Let θ be the acute angle between planes AOB and AOC. Given that $\cos^2\theta = \frac{m}{n}$ for relatively prime positive integers m and n, compute $100m + n$.

Problem 4. Let $A_0A_1 \dots A_{11}$ be a regular 12-gon inscribed in a circle with diameter 1. For how many subsets $S \subseteq \{1, \dots, 11\}$ is the product

$$\prod_{s \in S} A_0A_s$$

equal to a rational number? (The empty product is declared to be 1.)

Problem 5. Let a, b, c be positive integers and p be a prime number. Assume that

$$a^n(b + c) + b^n(a + c) + c^n(a + b) \equiv 8 \pmod{p}$$

for each nonnegative integer n. Let m be the remainder when $a^p + b^p + c^p$ is divided by p, and k the remainder when m^p is divided by p^4. Find the maximum possible value of k.

Time limit: 40 minutes
Maximum score is 42 points