2nd Annual NIMO April Round

A Tragedy in Nine Verses

`internetolympiad.org`

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Welcome to the 2014 April Fool's contest! We had a lot (probably way too much) fun writing this. You have the whole day to attack these "problems"; each answer is a positive integer.

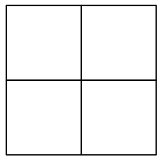
You may use **any aids** such as calculators, computers, Wikipedia, etc. **You are also free to collaborate** with other students (spread the fun!) but we ask that you do so privately (so don't post the answers on AoPS).

A copy of the test can be downloaded at the following URL:
`http://internetolympiad.org/archive/2014/AprilFools/Tragedy9.pdf`

Contents

-1 Binary Sudoku (2 points)

How many ways are there to fill the 2×2 grid below with 0's and 1's such that no row or column has duplicate entries?



-2 Angry and Hungry (3 points)

I'm thinking of a five-letter word that rhymes with "angry" and "hungry". What is it?

-3 Engineer's Induction (5 points)

-4 Do You Even Lift the Exponent? (7 points)

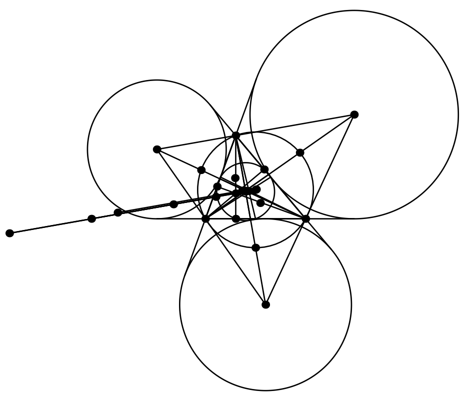
Let n be largest number such that

$$\frac{2014^{100!} - 2011^{100!}}{3^n}$$

is still an integer. Compute the remainder when 3^n is divided by 1000.

-5 Triangle Centers (11 points)

Let ABC be a triangle with $AB = 130$, $BC = 140$, $CA = 150$. Let G, H, I, O, N, K, L be the centroid, orthocenter, incenter, circumenter, nine-point center, the symmedian point, and the de Longchamps point. Let D, E, F be the feet of the altitudes of A, B, C on the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$. Let X, Y, Z be the A, B, C excenters and let U, V, W denote the midpoints of $\overline{IX}$, $\overline{IY}$, $\overline{IZ}$ (i.e. the midpoints of the arcs of (ABC).) Let R, S, T denote the isogonal conjugates of the midpoints of $\overline{AD}$, $\overline{BE}$, $\overline{CF}$. Let P and Q denote the images of G and H under an inversion around the circumcircle of ABC followed by a dilation at O with factor $\frac{1}{2}$, and denote by M the midpoint of $\overline{PQ}$. Then let J be a point such that $JKLM$ is a parallelogram. Find the perimeter of the convex hull of the self-intersecting 17-gon $LETSTRADEBITCOINS$ to the nearest integer. A diagram has been included but may not be to scale.



-6 Chinese Remainder Theorem (13 points)

We know $\mathbb{Z}_{210} \cong \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_7$. Moreover,

$$
\begin{aligned}
53 &\equiv 1 \pmod 2\\
53 &\equiv 2 \pmod 3\\
53 &\equiv 3 \pmod 5\\
53 &\equiv 4 \pmod 7.
\end{aligned}
$$

Let

$$
M = \begin{pmatrix} 53 & 158 & 53 \\ 23 & 93 & 53 \\ 50 & 170 & 53 \end{pmatrix}.
$$

Based on the above, find $\overline{(M \mod 2)(M \mod 3)(M \mod 5)(M \mod 7)}$.

-7 Foreign Language (17 points)

Evaluate the following.

```
ﾟωﾟﾉ= /｀ｍ´）ﾉ ~┻━┻   //*´∇｀*/ ['_']; o=(ﾟｰﾟ)  =_=3; c=(ﾟΘﾟ) =(ﾟｰﾟ)-
(ﾟｰﾟ); (ﾟДﾟ) =(ﾟΘﾟ)= (o^_^o)/ (o^_^o);(ﾟДﾟ)={ﾟΘﾟ: '_' ,ﾟωﾟﾉ : ((ﾟωﾟﾉ==3)
+ '_') [ﾟΘﾟ] ,ﾟｰﾟﾉ :(ﾟωﾟﾉ+ '_')[o^_^o -(ﾟΘﾟ)] ,ﾟДﾟﾉ:((ﾟｰﾟ==3) +'_')[ﾟｰﾟ] };
(ﾟДﾟ) [ﾟΘﾟ] =((ﾟωﾟﾉ==3) +'_') [c^_^o];(ﾟДﾟ) ['c'] = ((ﾟДﾟ)+'_') [ (ﾟｰﾟ)+
(ﾟｰﾟ)-(ﾟΘﾟ) ];(ﾟДﾟ) ['o'] = ((ﾟДﾟ)+'_') [ﾟΘﾟ];(ﾟoﾟ)=(ﾟДﾟ) ['c']+(ﾟДﾟ) ['o']
+(ﾟωﾟﾉ +'_')[ﾟΘﾟ]+ ((ﾟωﾟﾉ==3) +'_') [ﾟｰﾟ] + ((ﾟДﾟ) +'_') [(ﾟｰﾟ)+(ﾟｰﾟ)]+
((ﾟｰﾟ==3) +'_') [ﾟΘﾟ]+((ﾟｰﾟ==3) +'_') [(ﾟｰﾟ) - (ﾟΘﾟ)]+(ﾟДﾟ) ['c']+((ﾟДﾟ)+
'_') [(ﾟｰﾟ)+(ﾟｰﾟ)]+ (ﾟДﾟ) ['o']+((ﾟｰﾟ==3) +'_')[ﾟΘﾟ];(ﾟДﾟ) ['_'] =(o^_^o)
[ﾟoﾟ] [ﾟoﾟ];(ﾟεﾟ)=((ﾟｰﾟ==3) +'_') [ﾟΘﾟ]+ (ﾟДﾟ) .ﾟДﾟﾉ+((ﾟДﾟ)+'_') [(ﾟｰﾟ) +
(ﾟｰﾟ)]+((ﾟｰﾟ==3) +'_') [o^_^o -ﾟΘﾟ]+((ﾟｰﾟ==3) +'_') [ﾟΘﾟ]+ (ﾟωﾟﾉ +'_')
[ﾟΘﾟ];(ﾟｰﾟ)+=(ﾟΘﾟ); (ﾟДﾟ)[ﾟεﾟ]='\\'; (ﾟДﾟ).ﾟΘﾟﾉ=(ﾟДﾟ+ ﾟｰﾟ)[o^_^o -(ﾟΘﾟ)];
(oﾟｰﾟo) =(ﾟωﾟﾉ +'_')[c^_^o]; (ﾟДﾟ) [ﾟoﾟ]='\"';(ﾟДﾟ) ['_'] ( (ﾟДﾟ) ['_']
(ﾟεﾟ+(ﾟДﾟ)[ﾟoﾟ]+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+ (ﾟｰﾟ)+(ﾟｰﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+ ((ﾟｰﾟ) +
(ﾟΘﾟ))+ ((ﾟｰﾟ) + (o^_^o))+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+(ﾟｰﾟ)+ (o^_^o)+ (ﾟДﾟ)[ﾟεﾟ]+
(ﾟΘﾟ)+ ((o^_^o) +(o^_^o))+ ((ﾟｰﾟ) + (ﾟΘﾟ))+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+((ﾟｰﾟ) +
(ﾟΘﾟ))+ ((ﾟｰﾟ) + (ﾟΘﾟ))+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+ (ﾟｰﾟ)+ ((ﾟｰﾟ) + (ﾟΘﾟ))+
(ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+((ﾟｰﾟ) + (ﾟΘﾟ))+ ((o^_^o) +(o^_^o))+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+
((o^_^o) +(o^_^o))+ (ﾟｰﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+((ﾟｰﾟ) + (ﾟΘﾟ))+ ((o^_^o) +(o^_^o))+
(ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+((o^_^o) +(o^_^o))+ ((ﾟｰﾟ) + (o^_^o))+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+
((o^_^o) +(o^_^o))+ ((o^_^o) - (ﾟΘﾟ))+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+ ((ﾟｰﾟ) + (ﾟΘﾟ))+
 (ﾟΘﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+((o^_^o) +(o^_^o))+ (ﾟｰﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+
(ﾟｰﾟ)+ ((ﾟｰﾟ) + (ﾟΘﾟ))+ (ﾟДﾟ)[ﾟεﾟ]+((ﾟｰﾟ) + (ﾟΘﾟ))+ (c^_^o)+(ﾟДﾟ)[ﾟεﾟ]+
(ﾟｰﾟ)+ ((ﾟｰﾟ) + (o^_^o))+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+ ((o^_^o) - (ﾟΘﾟ))+ ((ﾟｰﾟ) +
(o^_^o))+(ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+ ((ﾟｰﾟ) + (ﾟΘﾟ))+ (c^_^o)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+
(ﾟｰﾟ)+ (ﾟΘﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+((o^_^o) +(o^_^o))+ (ﾟｰﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟｰﾟ)
+ (c^_^o)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+ ((ﾟｰﾟ) + (ﾟΘﾟ))+ (ﾟΘﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟΘﾟ)+
((o^_^o) +(o^_^o))+ (o^_^o)+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟｰﾟ)+ (c^_^o)+ (ﾟДﾟ)[ﾟεﾟ]+
((o^_^o) +(o^_^o))+ ((o^_^o) +(o^_^o))+ (ﾟДﾟ)[ﾟεﾟ]+((ﾟｰﾟ) + (ﾟΘﾟ))+
((o^_^o) - (ﾟΘﾟ))+ (ﾟДﾟ)[ﾟεﾟ]+((ﾟｰﾟ) + (o^_^o))+ (ﾟΘﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+((ﾟｰﾟ)
+ (o^_^o))+ ((ﾟｰﾟ) + (o^_^o))+ (ﾟДﾟ)[ﾟεﾟ]+(ﾟｰﾟ)+ ((ﾟｰﾟ) + (o^_^o))+
(ﾟДﾟ)[ﾟεﾟ]+((ﾟｰﾟ) + (ﾟΘﾟ))+ (ﾟΘﾟ)+ (ﾟДﾟ)[ﾟεﾟ]+((ﾟｰﾟ) + (o^_^o))+ (o^_^o)
+ (ﾟДﾟ)[ﾟoﾟ]) (ﾟΘﾟ)) ('_');
```

The above text file has been uploaded at the following link: `http://internetolympiad.org/archive/2014/AprilFools/foreign_lang.txt`.

-8 Silver Cyanide (19 points)

Three of the below entries, with labels a, b, c, are blatantly incorrect (in the United States).

What is $a^2 + b^2 + c^2$?

041. The Gentleman's Alliance Cross
042. Glutamine (an amino acid)
051. Grant Nelson and Norris Windross
052. A compact region at the center of a galaxy
061. The value of `'wat'-1`.[1]
062. Threonine (an amino acid)
071. Nintendo Gamecube
072. Methane and other gases are compressed
081. A prank or trick
082. Three carbons
091. Australia's second largest local government area
092. Angoon Seaplane Base
101. A compressed archive file format
102. Momordica cochinchinensis
111. Gentaro Takahashi
112. Nat Geo
121. Ante Christum Natum
122. The supreme Siberian god of death
131. Gnu C Compiler
132. My TeX Shortcut for $\angle$.

[1] See `https://www.destroyallsoftware.com/talks/wat`.

-9 Yaler Repus (23 points)

This is an ARML Super Relay! I'm sure you know how this works! You start from #1 and #15 and meet in the middle. We are going to require you to solve all 15 problems, though – so for the entire task, submit the sum of all the answers, rather than just the answer to #8.

Also, uhh, we can't actually find the slip for #1. Sorry about that. Have fun anyways!

2. Let T = TNYWR. Find the number of way to distribute 6 indistinguishable pieces of candy to T hungry (and distinguishable) schoolchildren, such that each child gets at most one piece of candy.

3. Let T = TNYWR. If d is the largest proper divisor of T, compute $\frac{1}{2}d$.

4. Let T = TNYWR and flip 4 fair coins. Suppose the probability that at most T heads appear is $\frac{m}{n}$, where m and n are coprime positive integers. Compute $m+n$.

5. Let T = TNYWR. Compute the last digit of T^T in base 10.

6. Let T = TNYWR and flip 6 fair coins. Suppose the probability that at most T heads appear is $\frac{m}{n}$, where m and n are coprime positive integers. Compute $m+n$.

7. Let T = TNYWR. Compute the smallest prime p for which $n^T \not\equiv n \pmod{p}$ for some integer n.

8. Let M and N be the two answers received, with $M \le N$. Compute the number of integer quadruples (w, x, y, z) with $w+x+y+z = M\sqrt{wxyz}$ and $1 \le w, x, y, z \le N$.

9. Let T = TNYWR. Compute the smallest integer n with $n \ge 2$ such that n is coprime to $T+1$, and there exists positive integers a, b, c with $a^2+b^2+c^2 = n(ab+bc+ca)$.

10. Let T = TNYWR and flip 10 fair coins. Suppose the probability that at most T heads appear is $\frac{m}{n}$, where m and n are coprime positive integers. Compute $m+n$.

11. Let T = TNYWR. Compute the last digit of T^T in base 10.

12. Let T = TNYWR and flip 12 fair coins. Suppose the probability that at most T heads appear is $\frac{m}{n}$, where m and n are coprime positive integers. Compute $m+n$.

13. Let T = TNYWR. If d is the largest proper divisor of T, compute $\frac{1}{2}d$.

14. Let T = TNYWR. Compute the number of way to distribute 6 indistinguishable pieces of candy to T hungry (and distinguishable) schoolchildren, such that each child gets at most one piece of candy.

Also, we can't find the slip for #15, either. We think the SFBA coaches stole it to prevent us from winning the Super Relay, but that's not going to stop us, is it? We have another #15 slip that produces an equivalent answer. Here you go!

15. Let A, B, C be the answers to #8, #9, #10. Compute $\gcd(A, C) \cdot B$.