April 1 - Solutions Sketch

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1. (2) $645 = \boxed{645}$.

2. (3) Assume there are k boys and $3k$ girls. At most $2013k$ edges exist. Hence $n \geq \frac{2013k}{3k} = \boxed{671}$ and this is achievable.

3. (3) Note that $o > l$. Since the numerator is shorter than the denominator, it has more l's. Hence $r < 1$. Also, $oll < 1$. So, $roll < 1$ and the answer is $\boxed{1}$.

4. (5) Since $b = 45$ and $c = 66$, we get $a = \boxed{69}$. In fact, the geo problem is vacuously true.

5. (5) By problem 4, $a + b + c = 180$. So, $b = s - b = 90 - b \implies b = \boxed{45}$.

6. (5) By problem 5, a, b, c are the sides of a triangle with $a + b + c = 180$ and $b = 45$. In particular, $a, b, c < 90$. Also, $c = \binom{2k}{2} \in \{1, 6, 15, 28, 45, 66, 91, \dots\}$. We have $90 > a = 135 - c$, so $c > 45$. Also, $c < 90$, so $c = \boxed{66}$, achieved when $n = -6$ and $k = 6$.

7. (7) Mod 3. Only $p = 3$ works and $\boxed{1781}$ is the answer after computation, with

$$N = 20 + 3^{68} = 278128389443693511257285776231781.$$

8. (13) Let the probability he flips a prime number $\frac{k}{2^{2010}}$, and note that $\gcd(k, 10) = 1$ by $k = \sum_{p \text{ prime}} \binom{2010}{p}$. Then $\frac{a}{b} = 0.01 \times \left(\frac{k}{2^{2010}} + \left(\frac{k}{2^{2010}} \right)^2 + \dots \right) = \frac{k}{100(2^{2010} - k)}$ so $100a + b = 100k + 100(2^{2010} - k) = 100 \cdot 2^{2010}$ and the answer is $\boxed{2017}$.

9. (17) Using problem 12, we have $645 - 671 + 1 - 69 + 45 - 66 + 1781 - 2017 + x - 358 + 80 - 1 = -1 \implies x = \boxed{629}$. In particular, love $= \frac{6}{29}$.

10. (11) The expression can be rewritten as

$$2^{80} \cdot 2^{4404} - \left(2^{52} - 2^{26} + 1\right) \left(2(2^{26} - 1)\right) \cdot 2^{2202} + 1 = \left(2^{2203} - 1\right) \left(2^{2281} - 1\right).$$

This is the product of two Mersenne primes. Their sum mod 1000 is $\boxed{358}$.

11. (23) It's "EIGHTY" upside-down (why else would we number right to left?), so $\boxed{80}$.

12. (7) $\boxed{1}$. Since there is an answer, we find $X_1 + X_3 + \dots + X_{11} = X_2 + X_4 + \dots + X_{10}$. Now 1 works and is achievable.