

Nobody is Missing Out
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Contest 0 Whenever
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1. Find the value of 645.
2. At a certain school, the ratio of boys to girls is 1 : 3. Suppose that:
 - Every boy has most 2013 distinct girlfriends.
 - Every girl has at least n boyfriends.
 - Friendship is mutual.

Compute the largest possible value of n .

3. Bored in an infinitely long class, Evan jots down a fraction whose numerator and denominator are both 70-character strings, as follows:

$$r = \frac{\text{looolooloolooloollllloollollollllloolllooolooloolooloolololoolololooooolllol}}{\text{loooloolollollollooooooolooloolloollloollolololooooolllooolollloool}}$$

If $o = 2013$ and $l = \frac{1}{50}$, find $\lceil roll \rceil$.

4. Let a, b, c be the answers to problems 4, 5, and 6, respectively. In $\triangle ABC$, the measures of $\angle A$, $\angle B$, and $\angle C$ are a, b, c in degrees, respectively. Let D and E be points on segments AB and AC with $\frac{AD}{BD} = \frac{AE}{CE} = 2013$. A point P is selected in the interior of $\triangle ADE$, with barycentric coordinates (x, y, z) with respect to $\triangle ABC$ (here $x + y + z = 1$). Lines BP and CP meet line DE at B_1 and C_1 , respectively. Suppose that the radical axis of the circumcircles of $\triangle PDC_1$ and $\triangle PEB_1$ pass through point A . Find $100x$.
5. Consider $\triangle \clubsuit \spadesuit$. Let \clubsuit , \spadesuit and \heartsuit be the answers to problems 4, 5, and 6, respectively. If the incircle of $\triangle \clubsuit \spadesuit$ touches \heartsuit at \heartsuit , find \heartsuit .
6. Let n and k be integers satisfying $\binom{2k}{2} + n = 60$. It is known that n days before Evan's 16th birthday, something happened. Compute $60 - n$.
7. Let p be the largest prime less than 2013 for which

$$N = 20 + p^{p^{p+1} - 13}$$

is also prime. Find the remainder when N is divided by 10^4 .

8. A person flips 2010 coins at a time. He gains one penny every time he flips a prime number of heads, but must stop once he flips a non-prime number. If his expected amount of money gained in dollars is $\frac{a}{b}$, where a and b are relatively prime, compute $\lceil \log_2(100a + b) \rceil$.
9. Haddaway once asked, "what is love?". The answer can be written in the form $\frac{m}{n}$, where m and n are positive integers such that $m^2 + n^2 < 2013$. Find $100m + n$.
10. There exist primes p and q such that

$$pq = 1208925819614629174706176 \times 2^{4404} - 4503599560261633 \times 134217730 \times 2^{2202} + 1.$$

Find the remainder when $p + q$ is divided by 1000.

11. <http://goo.gl/wVR25>
12. If X_i is the answer to problem i for $1 \leq i \leq 12$, find the minimum possible value of $\sum_{n=1}^{12} (-1)^n X_n$.