NIMO Monthly Contest

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Contest 5 8:00 PM – 8:30 PM EST

January 24, 2013

1. Tim is participating in the following three math contests. On each contest his score is the number of correct answers.

   - The Local Area Inspirational Math Exam consists of 15 problems.
   - The Further Away Regional Math League has 10 problems.
   - The Distance-Optimized Math Open has 50 problems.

   For every positive integer n, Tim knows the answer to the nth problems on each contest (which are pairwise distinct), if they exist; however, these answers have been randomly permuted so that he does not know which answer corresponds to which contest. Unaware of the shuffling, he competes with his modified answers. Compute the expected value of the sum of his scores on all three contests.

2. The cost of five water bottles is \$13, rounded to the nearest dollar, and the cost of six water bottles is \$16, also rounded to the nearest dollar. If all water bottles cost the same integer number of cents, compute the number of possible values for the cost of a water bottle.

3. In triangle ABC, $AB = 13$, $BC = 14$ and $CA = 15$. Segment BC is split into $n+1$ congruent segments by n points. Among these points are the feet of the altitude, median, and angle bisector from A. Find the smallest possible value of n.

4. The infinite geometric series of positive reals $a_1, a_2, \dots$ satisfies
$$1 = \sum_{n=1}^{\infty} a_n = -\frac{1}{2013} + \sum_{n=1}^{\infty} \text{GM}(a_1, a_2, \dots, a_n) = \frac{1}{N} + a_1$$
   where $\text{GM}(x_1, x_2, \dots, x_k) = \sqrt[k]{x_1 x_2 \cdots x_k}$ denotes the geometric mean. Compute N.

5. Compute the number of five-digit positive integers $\overline{vwxyz}$ for which
$$(10v + w) + (10w + x) + (10x + y) + (10y + z) = 100.$$

6. Tom has a scientific calculator. Unfortunately, all keys are broken except for one row: `1`, `2`, `3`, `+` and `-`. Tom presses a sequence of 5 random keystrokes; at each stroke, each key is equally likely to be pressed. The calculator then evaluates the entire expression, yielding a result of E. Find the expected value of E.

   (Note: Negative numbers are permitted, so `13-22` gives $E = -9$. Any excess operators are parsed as signs, so `-2-+3` gives $E = -5$ and `-+-31` gives $E = 31$. Trailing operators are discarded, so `2++-+` gives $E = 2$. A string consisting only of operators, such as `-++-+`, gives $E = 0$.)

7. For each integer $k \geq 2$, the decimal expansions of the numbers $1024, 1024^2, \dots, 1024^k$ are concatenated, in that order, to obtain a number X_k. (For example, $X_2 = 10241048576$.) If
$$\frac{X_n}{1024^n}$$
   is an odd integer, find the smallest possible value of n, where $n \geq 2$ is an integer.

8. Let $AXYZB$ be a convex pentagon inscribed in a semicircle with diameter AB. Suppose that $AZ - AX = 6$, $BX - BZ = 9$, $AY = 12$, and $BY = 5$. Find the greatest integer not exceeding the perimeter of quadrilateral $OXYZ$, where O is the midpoint of $\overline{AB}$.

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