

1. Compute the largest integer $N \leq 2012$ with four distinct digits.
2. A *normal magic square* of order n is an arrangement of the integers from 1 to n^2 in a square such that the n numbers in each row, each column, and each of the two diagonals sum to a constant, called the *magic sum* of the magic square. Compute the magic sum of a normal magic square of order 8.
3. A polygon $A_1A_2A_3 \dots A_n$ is called *beautiful* if there exist indices i, j , and k such that $\angle A_iA_jA_k = 144^\circ$. Compute the number of integers $3 \leq n \leq 2012$ for which a regular n -gon is beautiful.
4. When flipped, coin A shows heads $\frac{1}{3}$ of the time, coin B shows heads $\frac{1}{2}$ of the time, and coin C shows heads $\frac{2}{3}$ of the time. Anna selects one of the coins at random and flips it four times, yielding three heads and one tail. The probability that Anna flipped coin A can be expressed as $\frac{p}{q}$ for relatively prime positive integers p and q . Compute $p + q$.
5. In $\triangle ABC$, $AB = 30$, $BC = 40$, and $CA = 50$. Squares A_1A_2BC , B_1B_2AC , and C_1C_2AB are erected outside $\triangle ABC$, and the pairwise intersections of lines A_1A_2 , B_1B_2 , and C_1C_2 are P , Q , and R . Compute the length of the shortest altitude of $\triangle PQR$.
6. In $\triangle ABC$ with circumcenter O , $\angle A = 45^\circ$. Denote by X the second intersection of \overrightarrow{AO} with the circumcircle of $\triangle BOC$. Compute the area of quadrilateral $ABXC$ if $BX = 8$ and $CX = 15$.
7. The sequence $\{a_i\}_{i \geq 1}$ is defined by $a_1 = 1$ and

$$a_n = \lfloor a_{n-1} + \sqrt{a_{n-1}} \rfloor$$

for all $n \geq 2$. Compute the eighth perfect square in the sequence.

8. Compute the number of sequences of real numbers $a_1, a_2, a_3, \dots, a_{16}$ satisfying the condition that for every positive integer n ,

$$a_1^n + a_2^{2n} + \dots + a_{16}^{16n} = \begin{cases} 10^{n+1} + 10^n + 1 & \text{for even } n \\ 10^n - 1 & \text{for odd } n \end{cases}.$$