1. Dan the dog spots Cate the cat 50m away. At that instant, Cate begins running away from Dan at 6 $\frac{\text{m}}{\text{s}}$, and Dan begins running toward Cate at 8 $\frac{\text{m}}{\text{s}}$. Both of them accelerate instantaneously and run in straight lines. Compute the number of seconds it takes for Dan to reach Cate.

2. A permutation $(a_1, a_2, a_3, \ldots, a_{100})$ of $(1, 2, 3, \ldots, 100)$ is chosen at random. Denote by p the probability that $a_{2i} > a_{2i-1}$ for all $i \in \{1, 2, 3, \ldots, 50\}$. Compute the number of ordered pairs of positive integers (a, b) satisfying $\frac{1}{a^b} = p$.

3. For positive integers $1 \leq n \leq 100$, let
$$f(n) = \sum_{i=1}^{100} i|i - n|.$$
Compute $f(54) - f(55)$.

4. In $\triangle ABC$, $AB = AC$. Its circumcircle, Γ, has a radius of 2. Circle Ω has a radius of 1 and is tangent to Γ, $\overline{AB}$, and $\overline{AC}$. The area of $\triangle ABC$ can be expressed as $\frac{a\sqrt{b}}{c}$ for positive integers a, b, c, where b is squarefree and $\gcd(a, c) = 1$. Compute $a + b + c$.

5. If $w = a + bi$, where a and b are real numbers, then $\Re(w) = a$ and $\Im(w) = b$. Let $z = c + di$, where $c, d \geq 0$. If
$$\Re(z) + \Im(z) = 7,$$
$$\Re(z^2) + \Im(z^2) = 17,$$
then compute $\left|\Re\left(z^3\right) + \Im\left(z^3\right)\right|$.

6. A square is called *proper* if its sides are parallel to the coordinate axes. Point P is randomly selected inside a proper square S with side length 2012. Denote by T the largest proper square that lies within S and has P on its perimeter, and denote by a the expected value of the side length of T. Compute $\lfloor a \rfloor$, the greatest integer less than or equal to a.

7. Point P lies in the interior of rectangle $ABCD$ such that $AP + CP = 27$, $BP - DP = 17$, and $\angle DAP \cong \angle DCP$. Compute the area of rectangle $ABCD$.

8. The positive integer-valued function $f(n)$ satisfies $f(f(n)) = 4n$ and $f(n+1) > f(n) > 0$ for all positive integers n. Compute the number of possible 16-tuples $(f(1), f(2), f(3), \ldots, f(16))$.